7 *NRIAG Journal of Geophysics, Special Issue, PP. 85 - 95 (2004)*

LOCAL GEOID DETERMINATION AROUND LAKE NASSER

K. H. Zahran*, M. Rabah and H. Khalil*

ABSTRACT: The strategic importance of the area of Lake Nasser, where the High Dam exists, utilizes an accurate heights control for many aspects such as the determination of the lake level and the sedimentation rate at the bottom of the lake. GPS can be dedicating as a leveling control within this region only if the geoid undulations are accurately determined.

Computation of geoid heights involves processing of variations in gravity data, which reflect variations in the geoid surface. The computation relies on Stokes' integral. Geoid height accuracy is strongly influenced by how well the gravity data used in the computation represents the actual gravity field. Thus, 635 gravity points, on both sides of the lake, were used to describe the local gravity disturbance in the studied region.

As it is impractical to perform the integration over the entire surface of the Earth, a "remove restore" approach is often used with the long and short wavelength components of the geoid computed separately. The"removal" is done by computing the long wavelength component as gravity anomalies from a global spherical harmonic model. The short wavelength geoid height is then computed from local gravity disturbance. Finally the short and long wavelength components are added to give the required total geoid height values.

The final computed geoid surface ranging between 9.6 m to 10.9 m with a standard deviation of 0.29 m. The standard deviation of the residual after removing the regional trend is 0.18 m. Removing the linear trend from the residual improves the standard deviation of the residual to 0.13 m.

INTRODUCTION

Nasser Lake, which is impounded by the High Dam and extended 500 km in southern Egypt and northern Sudan along the main course of the River Nile, (Fig. 1), is considered to be of a great strategic importance for Egypt as it is the main source of water for the whole territory. Therefore, the High Dam Authority carries out an annual integrated research around the lake. Most of these researches require an accurate height reference control, which is not sufficient in these regions.

National Research Institute of astronomy and Geophysics, Cairo, Helwan, Egypt.

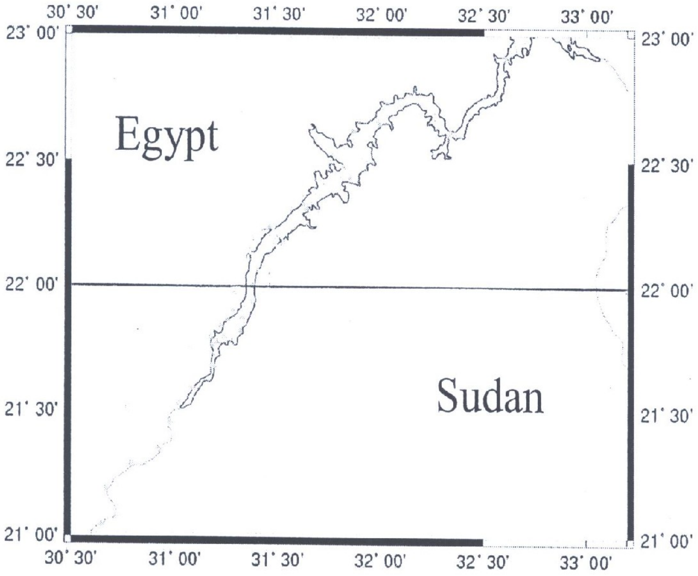

Fig. 1. Lake Nasser area.

Successful development of the Global Positioning system (GPS) permits the determination of positions very accurately in a terrestrial three-dimension over the world, but in many applications the system is used only for two-dimension positioning. This is due to the fact, that GPS provide us ellipsoidal height, which is geometric height; instead of orthometric height related to the mean see level with its physical meaning. Orthometric heights can be calculated from the potential difference of the reference equipotential surface and the actual point, while the potential difference can be determined with the combination of spirit leveling and gravity measurements. In order to convert ellipsoid height into a more useful orthometric height we need to know the geoid undulation at the station. The relation between ellipsoidal and orthometric heights can be seen in Figure (2). This means that by combining relative ellipsoidal heights Δh, from GPS and relative geoid heights ΔN, orthometric height differences can be determined ($\Delta H = \Delta h - \Delta N$).

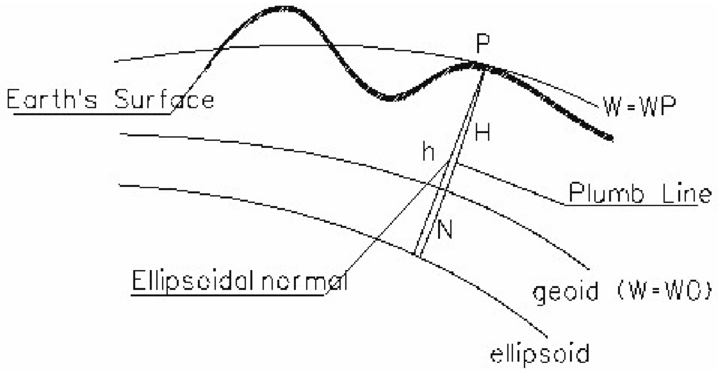

Fig. 2. The relation between orthometric and ellipsoidal heights.

Many advanced methods currently available for gravimetric geoid determination and the relative accuracy of the gravimetric geoid should meet at least the same accuracy level (Schwartz et al., 1987). Accurate gravimetric geoid determination requires optimal combination of different data types.

LOCAL GEOID DETERMINATION

Geoid can be determined from different types of input data. One of the possibilities to determine geoid is the gravimetric solution, which can be cutted out by the well-known Stockes formula for the geoid undulation relative to the reference ellipsoid

$$N = R/4\pi\gamma \iint_{\sigma} \Delta gi\ S\ (\psi i)\ d\sigma \qquad (1)$$

where S (ψ) is the Stockes function, R is the mean radius of the Earth and σ denotes the Earth‘s surface (see, e.g., Heiskanen and Moritz, 1967).

By using Eq. (1), the geoid undulation, representing the physical figure of the earth, can be determined from the gravity observations. This equation indicates that, we need to know the gravity values on the entire surface of the Earth for the geoid determination. It is impossible to get gravity data densely in a global distribution. Stock‘s integral can be modified when working in a local area by representing the surrounding gravity field by a global geopotential model such as EGM96 (Lemonie et al., 1996).

When a global geopotential model is available, Stokes' integral can be modified to integrate gravity anomalies over small cap σ (Kuroishi, 1995).

$$N = N_{GM} + R/4\pi\gamma \iint_{\sigma} (\Delta g - \Delta g_{GM})\, S(\psi)\, d\sigma \qquad (2)$$

Where NGM and ΔgGM are the geoid undulation and the gravity anomaly calculated by the geopotential model.

The effect of the topographic masses, which changes the potential field, has to be removed for accurate geoid computations. The present geoid computations are based on the"remove-restore" technique, therefore the geoid height computations were carried out according to the following equations:

$$N = N_{GM} + N\Delta g + Nh, \qquad (3)$$

Where the 1st term gives the contribution of the geopotential model, while the 2nd term gives the contribution of residual gravity anomalies with the effect of geopotential and terrain removed from the gravity anomaly. The 3rd term gives the indirect effect of the terrain reduction.

Application of spherical FFT in the geoid computations yield principally on the conversion of the residual gravity anomaly to the geoid undulations by using an improved multiband formulation of the original method (Foresberg and Sideris, 1993).

AVAILABLE DATA

The data used in the computations of the gravimetric geoid can be classified in three parts: geopotential model, terrestrial gravity and GPS data and a digital terrain model (DTM).

Equation (3) shows that geoid computations required the determination of the long wavelength of the potential field deduced from a geopotential model. Various geopotential models are available in assessing the contribution of geopotential model on geoid such as OSU91A (Rapp et al., 1991) or EGM96. Figures (3), (4) show the geoid undulations from OSU91A and EGM96 for the area of Lake Nasser and statistics of the geoid heights from both models are given in table (1). The EGM96 is the latest version of the geopotential model and a complete to degree and order 360. Thus EGM96 has been chosen as the geopotential model in the geoid computations in the current study.

Table 1. Statistics of the geoid heights.

	Max[m]	**Min[m]**	**Mean[m]**	**STD[m]**
OUS91A	8.0	11.5	9.38	0.73
EGM96	9.08	9.9	9.38	0.21

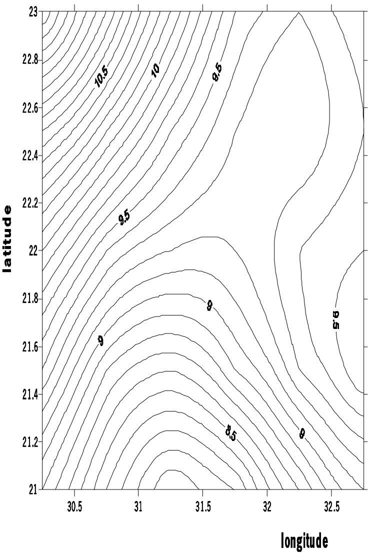

Fig. 3. The geoid undulation based on OSU91A geopotential model, geoid undulation: 8.0-11.5.

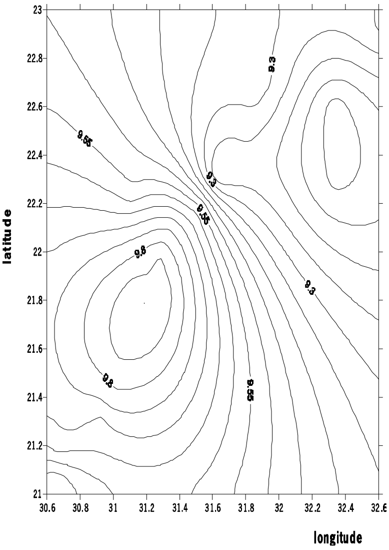

Fig. 4. The geoid undulation based on EGM96 geopotential model, geoid undulation: 9-9.9.

Because of the poor of gravity data around the Lake Nasser area, it has been decided to collect new gravity values around the lake. Two campaigns of data collections were carried out at the Sudanese and Egyptian parts of the Lake Nasser. Each of the gravity points was accompanied by rapid static GPS observations to determine its location and to use the GPS height with the gravity observation to determine the gravity disturbance. Thus, a number of 635 gravity GPS data distributed randomly, depending on the topography of the studied region around the lake, with spacing of about 1-3 km. Gravity values at the bottom of the lake have been interpolated from the land gravity data around the lake. Figure 5 shows the free air gravity anomaly as computed from the observed gravity data. The statistical properties of the free-air anomalies can be seen in table (2).

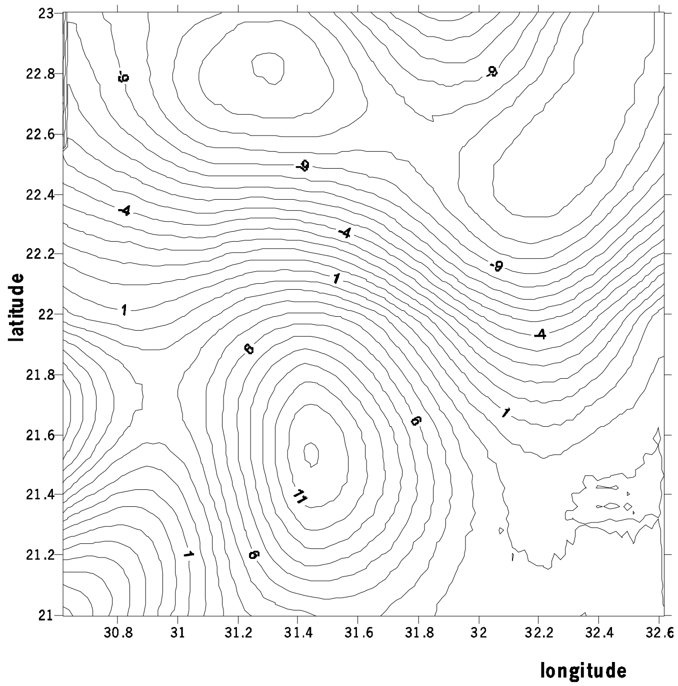

Fig. 5. Free air gravity anomaly of the lake Nasser area.

Table 2. Statistics of residual free-air gravity anomaly.

	Max [mgal]	**Min [mgal]**	**Mean [mgal]**	**STD [mgal]**
Residual anomalies	-13.2	12.0	-4.099	7.11

In additional to the reduction of the terrain to the gravity anomalies, indirect effect of the topography of the potential field is needed from a digital terrain model. The digital relief data base elevation ETOPO5 (NGDC, 1988) has been used to compute the terrain correction and their contribution of geoid prediction.

PRACTICAL COMPUTATIONS OF THE LOCAL GEOID USING FFT

Local geoid determination has been made using the FFT technique. Computation procedures can be summarized as follow:

- The gravity values have been used for computing the free air gravity anomalies, Using GPS heights for the classical terrain correction of the gravity data to determine the gravity disturbance and girded the data to a suitable grid points. The data are girded by minimum curvature spline.
- The reduced gravity anomalies together with the DTM are used for the geoid computations.
- The reduced geoid heights are carried out using the FFT technique for the Stoke's integral with the reduced gravity anomalies.
- Using the DTM the indirect effect on the geoid has been determined.
- The terrain effects of the terrain corrections on the geoid are computed using FFT for the Stok's formula.
- Finally, the geoid heights are carried out as a sum of geopotential model geoid heights obtained from EGM96.

RESULTS

Figure (6) represents the residual geoid undulation due to the local gravity effect; figure (7) represents the final geoid of the area of Lake Nasser. The major contribution of the final geoid is coming from the EGM96 geopotential model with values ranging from 9.1 m to 9.9 m and a standard deviation of 0.21 m. The standard deviation of the contribution from observed gravity data are 0.18 m. and the statistics of both of them are tabulated on table (3).

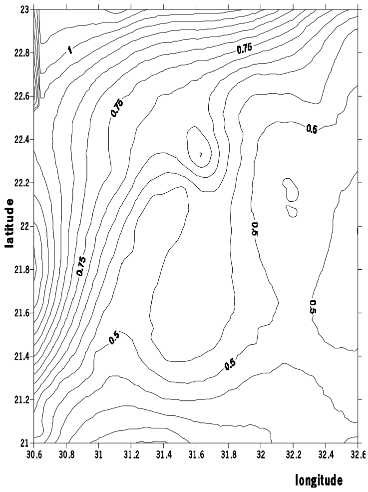

Fig. 6. Residual geoid undulation obtained from two dimensional spherical FFT.

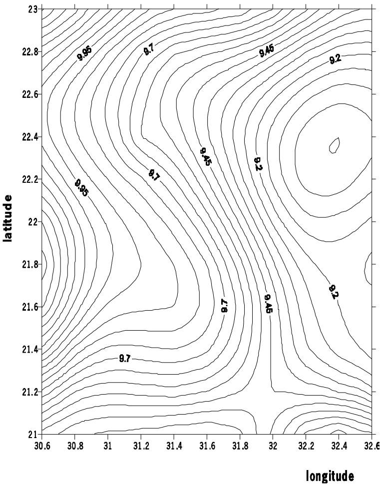

Fig. 7. Geoid map based on EGM96 geopotential model by means two dimensional spherical FFT.

Table 3. Statistics of the geoid heights.

	Max[m]	Min[m]	Mean[m]	STD[m]
computed	9.56	10.85	10.09	0.29
Residual	0.29	1.12	0.62	0.18

However, the maximum effect of the gravity data exceeds one meter in the north western part of the area. Generally, the effect of the gravity data increased toward the north, where less gravity data are available due to the increasing of the lake width toward the north. The final geoid surface ranging between 9.6 m to 10.9 m with a standard deviation of 0.29 m. The geoid surface shows a NW-SE trend and a decrease of the geoid height towered the east. Difference of about 1.2 m from the west side of the lake to the east side of the lake. In order to improve this gravimetric solution, the long wavelength part of the residuals was removed from the original gravimetric geoid. The linear trend was filtered out from the dataset of the residuals, and the gravimetric geoid was corrected with these values. The linear trend of the gravimetric geoid can be seen in (fig. 9) and the residual after removing this trend are given in figure (8), while comparison between the residual before and after removing the linear trend can be found in table (4).

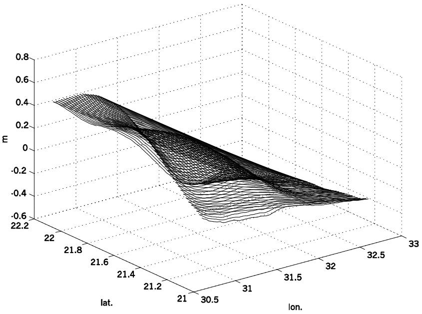

Fig. 8. The residual after removing the linear trend.

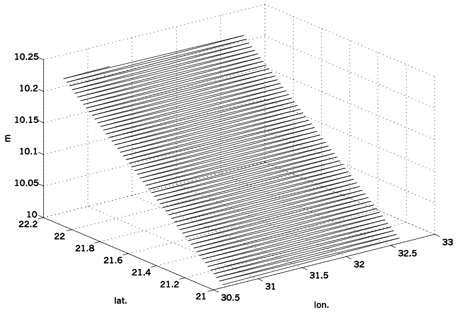

Fig. 9. Linear trend of the residual

Table 4. Comparison of the residual before and after the removal of the linear trend.

with linear trend [m]				without linear trend [m]			
Min.	Max.	Mean	SD	Min.	Max.	Mean	SD
0.29	1.12	0.62	0.18	0.21	0.86	0.53	0.13

CONCLUSION

In the current study a gravimetric geoid was determined at the area of Lake Nasser in order to permit the GPS heights to be used as a height control. Computations were made by involving the observed free air gravity anomaly with the geopotential model by two dimensional spherical FFT.

The obtained geoid values have a standard deviation of 0.29 m and that of the residual is 0.18m. Removing the linear trend from the residual improves it significantly. Increasing the effect of the gravity anomaly toward the north may be due to the lack of the data in the north as a result of the width increasing of the lake in the north.

It is demonstrated that this gravimetric geoid meets the requirements of most geodetic applications, and can be assert the application of GPS in the field of orthometric height determination.

GPS/leveling points are needed to improve and evaluate the current gravimetric geoid for practical orthometric heights determinations.

ACKNOWLEDGEMENT

Gravity and GPS data have been collected within the frame of two research trips funded by the High Dam authority.

REFERENCES

Forsberg,R. and M.G.Sideris, 1993. Geoid computation by the multi-band spherical FFT approach, Manuscr. Geod., 18, 82-90.

Heiskanen, W.A. and H. Moritz, Physical geodesy, 1967. 364pp., Freeman and company, San Francisco.

Kuroishi,Y., 1995. Precise gravimetric determination of geoid in the vicinity of Japan, Bull. GSI, 41, 9-11.

Lemonie, F.G., D.E. Smith, L.Kunz, R. Smith, E.C. Pavilis, N.K. Pavilis, S.M. Klosko, D.S. Chinn, M.H. Torrence, R.G. Williamson, C.M. Cox, K.E. Rachlin, Y.M. Ming, S.C. Kenyon, R. Salman, R. Trimmer, R.H. Rapp and R.S. Nerem, 1996. The development of the NASA GSFC and NIMA joint geopotential model, Gravity, Geoid and Marine Geodesy, International Symposium, No. 117, 461-469.

NGDC, 1988 ETOPO5 Data Announcement 88-MGG-02, Digital Relief of the Surface of the Earth. NOAA, National Geophysical Data Center, Boulder, Colorado.

Rapp, R . H., T. H. Wang, and N. K. Pavlis, 1991. The Ohio State 1991 geopotential and sea surface topography harmonic coefficient models, Report No.410, Dept. of Geodetic Science and surveying, Ohio State University.

Schwartz, K. P., M. G. Sideris and R. Forsberg, 1987. Orthometric heights without leveling, J. Surv. Eng., 113, No. 1, 28-40.